Written Exam for the B.Sc. in Economics 2010-I

## Micro 3

Final Exam

January 2010
(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

## PLEASE ANSWER ALL QUESTIONS BELOW. PLEASE EXPLAIN YOUR ANSWERS.

1. (a) Find all Nash equilibria in the following game

|  | L | R |
| :--- | :--- | :--- |
| U | $0,-1$ | $-1,2$ |
| D | 4,2 | $-2,0$ |

(b) Consider the following normal-form game:

|  | $t_{1}$ | $t_{2}$ | $t_{3}$ |
| :---: | :---: | :---: | :---: |
| $s_{1}$ | $0, x$ | 0,3 | 1,2 |
| $s_{2}$ | $1,2 x$ | 2,4 | $-1,0$ |

i. Under what range of values of parameter $x$ is strategy $t_{1}$ strictly dominated?
ii. Assume that $x$ is such that $t_{1}$ is strictly dominated. Find all Nash equilibria of this game.
(c) Consider the extensive-form game represented by the game tree on Figure 1:


Figure 1
i. How many subgames are there in this game? Find all subgame perfect Nash equilibria
ii. Find all pure-strategy Nash equilibria (Hint: rewrite the game in a normal form). Comment based on the game in question where does the difference between your answers to (i) and (ii) (if any) come from.
(d) Can a strictly dominated strategy be part of SPNE? If yes, suggest an example. If no, explain why not. (Be short and precise).
2. Two firms, firm 1 and firm 2, are competing on the market in Cournot-fashion. The inverse demand function in this market is given by

$$
P=a-\left(q_{1}+q_{2}\right),
$$

where $q_{i}, i=1,2$, are the output levels set by the firms, $a>0$, and $P$ is the price. Both firms have constant marginal costs $c>0$ (where $a>c$ ). The profit of firm $i$ is thus

$$
\pi_{i}\left(q_{1}, q_{2}\right)=\left(a-\left(q_{1}+q_{2}\right)\right) q_{i}-c q_{i}, \quad i=1,2 .
$$

The owner of firm 1 manages her firm herself. She chooses $q_{1}$ to maximize the profit of her firm $\pi_{1}\left(q_{1}, q_{2}\right)$. The owner of firm 2 hires a manager instead of managing the firm herself. She
provides the manager with an incentive package based on firm 2's profits and firm 2 's sales $q_{2}$. More precisely, the payment to the manager, which equals his payoff is given by

$$
u_{m}\left(q_{1}, q_{2}, \alpha\right)=0.1 *\left[\pi_{2}\left(q_{1}, q_{2}\right)+\alpha q_{2}\right],
$$

where $\alpha$ is the relative importance of firm 2's sales level in the manager's incentive package. The owner of firm 2 chooses $\alpha$ in order to maximize her income, i.e. firm 2's profit minus the payment to the manager

$$
\begin{aligned}
& \pi_{2}\left(q_{1}, q_{2}\right)-u_{m}\left(q_{1}, q_{2}, \alpha\right) \\
= & \pi_{2}\left(q_{1}, q_{2}\right)-0.1 *\left[\pi_{2}\left(q_{1}, q_{2}\right)+\alpha q_{2}\right]
\end{aligned}
$$

The timing of the game is as follows: First, the owner of firm 2 chooses $\alpha$ (i.e., the composition of the incentive package). Then, both the owner of firm 1 and the manager of firm 2 observe $\alpha$ and simultaneously choose $q_{1}$ and $q_{2}$, respectively.
(a) What are the levels of output chosen by the owner of firm 1 and the manager of firm 2 in the second stage of the game? How do they depend on $\alpha$ ? Provide an intuition for your answer.
(b) Find the level of $\alpha$ chosen by the owner of firm 2 in the subgame perfect equilibrium and find the levels of output in the subgame perfect equilibrium? Comment on how the strategic market position of firm 2 is affected by hiring a manager with an incentive package.
3. Consider the following signalling game.

(a) Find a pooling perfect Bayesian equilibrium or show that there is none.
(b) Find a separating perfect Bayesian equilibrium or show that there is none.
4. Two brothers, Anders and Thomas, are bargaining about the way of sharing the 500 -gram cake their mother has baked for them. Anders' utility from consuming $x_{A}$ grams of cake is given by

$$
u_{A}\left(x_{A}\right)=x_{A} .
$$

Thomas' utility from getting $x_{T}$ grams of cake is given by

$$
u_{T}\left(x_{T}\right)=2 x_{T} .
$$

Their mother told them that if they fail to reach an agreement, she will give the cake to their neighbors, so that neither of brothers receives anything.
(a) Represent the situation as a bargaining problem, i.e. draw the sets $X$ and $U$, and mark the disagreement points. Describe the efficient allocations.
(b) Determine the Nash bargaining solution of the game.
(c) Assume now that once Anders has eaten enough of the cake, he is satiated and does not enjoy more cake. More precisely, he does not get any extra utility from consuming more than 300 grams of cake, that is, his utility is now given by

$$
u_{A}\left(x_{A}\right)=\left\{\begin{array}{ll}
x_{A}, & \text { if } x_{A} \leq 300 \\
300, & \text { if } x_{A}>300
\end{array} .\right.
$$

Thomas' utility is still $u_{T}\left(x_{T}\right)=2 x_{T}$. Draw the modified set $U$, and apply the Nash bargaining solution axioms to your answer in (b) to find the Nash bargaining solution of this problem.

